Let $u = 7 + x^4$. Then $du = 4x^3\,dx$ and $x^3\,dx = \frac{1}{4}\,du$,

so $\int x^3(7+x^4)^7\,dx = \int u^7\left(\frac{1}{4}\,du\right) = \dfrac{1}{4}\dfrac{u^8}{8} + C = \frac{1}{32}(7+x^4)^8 + C.$